INFLUENCE OF THERMAL SOFTENING ON THE VISCOPLASTIC SHELL COLLAPSE PROCESS

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More and more attention has been paid in recent years to investigating the problem of cumulative energy during the collapse of incompressible shells of different geometry. Results of analyzing thermal dissipation processes during the collapse of viscous shells are displayed in [1-3], of rigidly plastic in [4], and viscoplastic in [5-10]. The possibility of reaching significant temperature gradients and the occurrence of phase transformations (melting, evaporation) in layers of the shell material abutting on the interior surface is the main deduction of the research performed. The investigations in [1-10] are carried out within the framework of phenomenological plasticity theory. It is considered here that irreversible strains occur in the material element when the stress state of this element reaches the limit surface (flow surface). This means that utilization of the viscosity η and yield point Y constants in the analysis implicitly assumes operation with average values of these quantities.

There is currently no unique treatment for the solution of the problem of cumulative thermal energy with thermal softening of the shell material taken into account. Thus, it is proposed to take account of the temperature dependence of viscosity in [10] by separating out the "heated" sublayer of the shell with an appropriate diminished value of the material viscosity. The dependence of Y and η on the temperature is not taken into account here in the equations of motion. In [11] this question is analyzed in application to the problem of collapse of a spherical viscoplastic shell. It is shown that by taking account of the thermal softening effects the problem of determining the velocity of interior surface motion for a spherical shell is reduced to solving an integrodifferential equation. The expression for the internal energy is obtained on the basis of its association with the energy expended in compressing the shell.

Let us examine an alternative method of obtaining a closed system of equations and let us also discuss the possible methods of taking account of thermal softening effects in problems of incompressible viscoplastic shell collapse. Let a cylindrical or spherical shell be deformed under the action of a constant external pressure P (a and b are the inner and outer radii of the shell). The shell material is assumed homogeneous, isotropic, incompressible, and subject to the governing relationships of a viscoplastic medium.

The equations of continuity, motion, heat influx, and the governing relationships with the assumptions made taken into account are written in the form

$$\frac{\partial}{\partial r}(r^{\nu}v)=0; \tag{1}$$

$$\rho\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}\right)=\frac{\partial\sigma_r}{\partial r}+\frac{\nu}{r}(\sigma_r-\sigma_\theta); \tag{2}$$

$$\frac{\partial e}{\partial t}+v\frac{\partial e}{\partial r}=\frac{c\varkappa}{r^{\nu}}\frac{\partial}{\partial r}\left(r^{\nu}\frac{\partial T}{\partial r}\right)+\Phi; \tag{3}$$

$$\sigma_r-\sigma_\theta=Y+2\eta\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right). \tag{4}$$

Here t is time; r, Euler coordinate; v, velocity of radial motion; ρ, density; e, internal energy; T, temperature; c, specific heat; $\varkappa=\lambda/(c\rho)$, thermal diffusivity; ν, geometry parameter equal to 1 and 2 for cylindrical and spherical symmetry, respectively; σ_r and σ_θ, principal stresses, where $\sigma_z=(\sigma_r+\sigma_\theta)/2$ for $\nu=1$, the plane-strain state, and $\sigma_\varphi=\sigma_\theta$ for $\nu=2$.

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The initial and boundary conditions of the problem have the form

$$t=0:\ r(0)=r_0,\ v(r)=0,\ T(r)=T_0,\ e(r)=e_0,\ Y(r)=Y_0,\ \eta(r)=\eta_0;$$
$$r=a:\ v=\dot{a},\ \sigma_r=0,\ \partial T/\partial r=0;\ r=b:\ \sigma_r=-P,\ \partial T/\partial r=0, \tag{5}$$

where $\dot{a} = da/dt$ is the velocity of interior surface motion; the subscript 0 refers to initial values of the quantities; and the dot denotes differentiation with respect to time here and below.

The function Φ in the right side of (3) characterizes the dissipation intensity, i.e., the rate of heat formation per unit mass of substance because of the transition of mechanical into thermal energy. Taking into account that the intensity of internal dissipation is proportional to the strain rate intensity, we can write

$$\Phi=\sigma_i\dot{\varepsilon}_i/\rho. \tag{6}$$

Here σ_i and $\dot{\varepsilon}_i$ are the stress and strain rate intensities; $\sigma_i=\frac{(\sqrt{3})^\nu}{\nu+1}(\sigma_r-\sigma_\theta)$, $\dot{\varepsilon}_i=-\frac{\nu(\nu+1)}{(\sqrt{3})^\nu}\frac{v}{r}$. Taking account of these latter relationships, (6) is manipulated into

$$\Phi=-\frac{\nu Y}{\rho}\frac{v}{r}+2\nu(\nu+1)\frac{\eta}{\rho}\left(\frac{v}{r}\right)^2. \tag{7}$$

Therefore, two sources of internal dissipation exist for the rheological material models under consideration, where the first is associated with internal structure changes caused by plastic deformations and the second with rheological or viscous properties.

We introduce the average pressure $p = -(\sigma_r + \nu\sigma_\theta)/(\nu + 1)$ to determine the law of shell motion. Taking account of this equality, the expression for the radial stresses can be represented in the form

$$\sigma_r=-p+\frac{\nu}{\nu+1}(\sigma_r-\sigma_\theta). \tag{8}$$

Using the relationships (4) and (8), and taking the value of the first integral in (1) into account,

$$v=\dot{a}(a/r)^\nu, \tag{9}$$

we convert the equation of motion (2):

$$\rho\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}\right)=-\frac{\partial p}{\partial r}+\frac{\nu}{\nu+1}\frac{\partial Y}{\partial r}+\nu\frac{Y}{r}-2\nu\frac{\dot{a}a^\nu}{r^{\nu+1}}\frac{\partial\eta}{\partial r}. \tag{10}$$

In this case the impulse of the viscous forces differs from zero. Therefore, the influence of viscosity on the process will be felt not only in terms of the boundary conditions, as occurs in the model with constant viscosity and yield point. The boundary conditions are determined from (8) with (4), (5), and (9) taken into account and have the following form relative to the average pressure:

$$p(a,t)=\frac{\nu}{\nu+1}Y(a,t)-2\nu\frac{\eta(a,t)\dot{a}}{a},$$
$$p(b,t)=P+\frac{\nu}{\nu+1}Y(b,t)-2\nu\frac{\eta(b,t)\dot{a}a^\nu}{b^{\nu+1}}. \tag{11}$$

Integrating (10) with respect to the radius between a and r with (9) and the boundary condition (11) taken into account for $r = a$, we obtain the pressure distribution

$$p(r,t)=-\rho\left\{(\ddot{a}a^\nu+\nu\dot{a}^2a^{\nu-1})f-\frac{\dot{a}^2}{2}\left[1-\left(\frac{a}{r}\right)^{2\nu}\right]\right\}-2\nu\frac{\eta(a,t)\dot{a}}{a}+$$
$$+\frac{\nu}{\nu+1}Y(r,t)+\nu\int_a^r\frac{Y(r',t)}{r'}dr'-2\nu\dot{a}a^\nu\int_a^r\frac{\partial\eta(r',t)}{\partial r}\frac{1}{r'^{(\nu+1)}}dr',$$

where $f=\begin{cases}\ln(r/a) & \text{for} \quad \nu=1,\\ (r-a)/(ar) & \text{for} \quad \nu=2.\end{cases}$ Hence, taking (11) into account for r = b, we write the law of shell inner surface motion

$$\ddot{a}=\left\{\frac{1}{2Fa^{\nu}}\left[1-\left(\frac{a}{b}\right)^{2\nu}\right]-\frac{\nu}{a}\right\}\dot{a}^{2}-\frac{2\nu\eta(a,t)\dot{a}}{\rho F a^{\nu+1}}\left[1-\frac{\eta(b,t)}{\eta(a,t)}\left(\frac{a}{b}\right)^{\nu+1}\right]+ \\ +\frac{1}{\rho F a^{\nu}}\left\{\nu\int\limits_{a}^{b}\left[\frac{Y(r,t)}{r}-\frac{2\dot{a}a^{\nu}}{r^{\nu+1}}\frac{\partial\eta(r,t)}{\partial r}\right]dr-P\right\}. \quad (12)$$

Here $F=\begin{cases}\ln(b/a) & \text{for} \quad \nu=1,\\ (b-a)/(ab) & \text{for} \quad \nu=2;\end{cases}$ Y(r, t) and η(r, t) in the last equation are functions of the temperature

$$Y=Y_{0}\varphi(T),\ \eta=\eta_{0}g(T), \quad (13)$$

for whose determination we must rely upon the heat influx differential equation [φ(T) and g(T) are known dimensionless functions describing the dependence of the yield point and viscosity on the temperature].

Substituting relationships (7) and (9) successively into (3) and taking into account that e = cT, we obtain

$$\frac{\partial T}{\partial t}+\dot{a}\left(\frac{a}{r}\right)^{\nu}\frac{\partial T}{\partial r}=\frac{\varkappa}{r^{\nu}}\frac{\partial}{\partial r}\left(r^{\nu}\frac{\partial T}{\partial r}\right)-\nu\frac{Y(r,t)\dot{a}a^{\nu}}{c\rho r^{\nu+1}}+2\nu(\nu+1)\frac{\eta(r,t)\dot{a}^{2}a^{2\nu}}{c\rho r^{2(\nu+1)}}. \quad (14)$$

Boundary conditions (12)-(14) for the problem are converted to the form

$$t=0:\ r(0)=r_{0},\ \dot{a}=0,\ T(r)=T_{0},\ Y(r)=Y_{0},\ \eta(r)=\eta_{0}; \\ r=a:\ \partial T/\partial r=0;\ r=b:\ \partial T/\partial r=0. \quad (15)$$

System (12)-(15) is closed and describes the nonlinear vibrations of an incompressible viscoplastic shell. Taking into account that $\dot{a}$ = 0 and $\ddot{a} \le 0$ for t = 0, we have $P \ge Y_*$, $Y_* = \nu Y_0 \ln(b_0/a_0)$ (Y_* is the effective material yield point under compression).

The approach considered results in a different form from [11] for the dynamic component describing the rheological (viscous) effects and the equations of viscoplastic shell motion. This is essential also in the case of practical importance examined in [1, 2], where the process of thermal energy accumulation was investigated during collapse of a viscous cylindrical shell. Even in the problem of viscous shell collapse it is evident that taking account of the temperature dependence of the material viscosity will result in a difference in the form of the dynamic component describing the rheological effects. In a particular case ($\eta = \eta_0$ = const) Eq. (12) is converted to that obtained in [1, 2]. The viscosity can here be interpreted not only as the average characteristic of the material but also as a quantity governing the rheology of the behavior of the internal layer of the shell material.

An alternate method of perfecting the theory taking account of the effect of thermal material softening is associated with using concepts of average characteristics $\langle Y\rangle = Y_0\varphi(\langle T\rangle)$, $\langle\eta\rangle = \eta_0 g(\langle T\rangle)$ [$\langle T\rangle = \langle T(t)\rangle$ is the average temperature (over the layer)]. The equation of motion of the shell inner surface (12) is then converted to that examined in [1-10]. Let us emphasize that construction of the solution during operation with ⟨Y⟩ and ⟨η⟩ assumes averaging of the temperature distribution T(r) over the appropriate coordinate r in each time step Δt.

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RESONANCE BENDING WAVES IN A CYLINDRICAL SHELL UNDER A MOVING RADIAL LOAD

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Analysis of axisymmetric wave processes in infinite cylindrical systems shows [1, 2] that critical velocities of motion exist in the axial direction of the surface load that forms resonance perturbations. If the load velocity agrees with the "rod" velocity ($c_s = \sqrt{E/\rho}$), longwave resonance of the longitudinal vibrations is realized. Another critical velocity corresponds to the medium-wave part of the spectrum and to the minimum of the dispersion curve of the first mode.

The asymptotic of resonance wave growth in shells is obtained in [1-3] for comparatively large values of the time ($t \to \infty$). Applicability of the asymptotic solution for finite values of the time is investigated only for low-frequency longitudinal resonance processes [2, 4, 5]. A bending resonance wave asymptotic is obtained below for a different kind of load and its applicability is clarified for quantitative estimates in systems of bounded length. The kind of load is determined for which the perturbations grow substantially more rapidly than in other cases.

Formulation of the Problem

Shell dynamics is described by the linear equations of classical Kirchhoff-Love theory:

$$\ddot{u} = u''_x + \nu w'_x, \quad \ddot{w} = -\nu u'_x - w - \varepsilon u^{\mathrm{IV}}_x + Q/h, \quad \varepsilon = h^2/12, \tag{1}$$

where u and w are the shell displacements in the axial x and radial directions; h is the shell thickness, and Q is the acting load. Taken as units of measurement are $c = \sqrt{E/[\rho(1-\nu^2)]}$ the speed of sound in a thin plane (E is Young's modulus, ν is the Poisson ratio), R is the

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